Some Improved Ratio Estimators Using Known Values of Some Population Parameters

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Abstract

In this study, three modified ratio estimators were proposed which were compared with some existing ratio estimators depending on the value of α being considered. The conditions attached to these proposed modified ratio estimators are that: $0 < \alpha < 1$, $(\frac{\bar{X} + \rho_{xy}}{\bar{X}}) \geq 1$, $(\frac{\bar{X} + c_x}{\bar{X}}) \geq 1$ and $(\frac{\bar{X} + \beta_{xy}}{\bar{X}}) \geq 1$ while those of the existing ones are: $(\frac{\bar{X}}{\bar{X} + \rho_{xy}}) < 1$, $(\frac{\bar{X}}{\bar{X} + c_x}) < 1$ and $(\frac{\bar{X}}{\bar{X} + \beta_{xy}}) < 1$ which are in opposite direction to the proposed modified ratio estimators. Two data sets (populations I and II) are used to justify this work using their derived mean square error (mse) and the proposed modified ratio estimators $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$ were found to be better and more efficient than those existing ratio estimators. However, for populations I and II, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$ are preferred respectively because they had the least mean square error (mse). In this study, whenever $\bar{X} < \bar{Y}$, the range values of α was found to be wider (i.e. $0.1 \leq \alpha \leq 0.6$) than when $\bar{X} > \bar{Y}$ (i.e. $0.1 \leq \alpha \leq 0.2$). Hence, these proposed modified ratio estimators are recommended for future usage in sample surveys.

Keywords: Ratio, Estimator, Modified, Mean Square Error

1.0 Introduction

Let N and n be the population and sample sizes respectively, $\bar{X}$ and $\bar{Y}$ be the population means for the auxiliary variable (X) and the variable of interest (Y) respectively, $\bar{x}$ and $\bar{y}$ be their respective sample means based on the sample drawn. If the correlation between the study variable y and the auxiliary variable x is positive (high), the ratio method of estimation is quite effective but if the correlation between the study variable y and the auxiliary variable x is negative (low), the product method of estimation is quite effective. This study is interested only in the ratio method of estimation when the correlation between the study variable y and the auxiliary variable x is positive (high).

Then classically [1 - 4],

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}} \bar{X} \tag{1}$$

and

$$mse(\bar{y}_r) = (\frac{N-n}{Nn}) \bar{Y}^2 [c^2{}_x + c^2{}_y - 2\rho_{xy} c_x c_y)] \tag{2}$$

In sample surveys, supplementary information is often used to increase the precision of estimators [5–8]. Many authors have used auxiliary information for improved estimation of population mean of study variable y [9–12].

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2.0 Methodology

2.1 On Khoshnevisan et al. Ratio Family Estimators

Suppose a pair (x_i, y_i) (i=1, 2, ….., n) of observations are taken on n units sampled form N population units using simple random sampling without replacement scheme, $\bar{X}$ and $\bar{Y}$ are the population means for the auxiliary variable (X) and variable of interest (Y), and $\bar{x}$ and $\bar{y}$ are the sample means based on the sample drawn.

A family of estimators was suggested [1] and expressed as:

$$t = \bar{y}[\frac{a\bar{X}+b}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{X}+b)}]^g \tag{3}$$

Where $a \neq 0$, b are either real numbers or a functions of the known parameters of the auxiliary variable x such as standard derivation σ_x, coefficient of variation, c_x, Skewness $\beta_{1(x)}$, Kurtosis $\beta_{2(x)}$ and correlation coefficient ρ_{xy}.

(i) when α = 0, a = 0 =b, g = 0, the mean per unit estimator, $t_0 = \bar{y}$ with

$$mse(t_o) = (\frac{N-n}{Nn})\bar{Y}^2 c^2{}_y \tag{4}$$

(ii) when α=1 a=1, b= 0, g=1, the usual ratio estimator, $t_1 = \frac{\bar{y}\bar{X}}{\bar{x}}$ with

$$mse(t_1) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x + c^2{}_y - 2\rho_{xy}c_x c_y] \tag{5}$$

(iii) when α=1 a=1, b= ρ_{xy} ,g=1, the ratio estimator, $t_2 = \bar{y}[\frac{\bar{X}+\rho_{xy}}{\bar{x}+\rho_{xy}}]$ with

$$mse(t_2) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_y + c^2{}_x(\frac{\bar{X}}{\bar{X}+\rho_{xy}}) - 2(\frac{\bar{X}}{\bar{X}+\rho_{xy}})\rho_{xy}c_x c_y)] \quad [13] \tag{6}$$

(iv) when α=1 a=1, b=c_x, g=1, the ratio estimator, $t_3 = \bar{y}[\frac{\bar{X}+c_x}{\bar{x}+c_x}]$ with

$$mse(t_3) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_y + c^2{}_x(\frac{\bar{X}}{\bar{X}+c_x}) - 2(\frac{\bar{X}}{\bar{X}+c_x})\rho_{xy}c_x c_y)] \quad [14] \tag{7}$$

(v) when α=1, a=1, b= β_{xy} , g=1, the ratio estimator, $t_4 = \bar{y}[\frac{\bar{X}+\beta_{xy}}{\bar{x}+\beta_{xy}}]$ with

$$mse(t_4) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_y + c^2{}_x(\frac{\bar{X}}{\bar{X}+\beta_{xy}}) - 2(\frac{\bar{X}}{\bar{X}+\beta_{xy}})\rho_{xy}c_x c_y)] \tag{8}$$

2.2 On Adewara et al. Modified Ratio Estimators

Adopting Adewara [5], $t^*{}_1 = \frac{\bar{y}^* \bar{X}}{\bar{x}^*}$, $t^*{}_2 = \bar{y}^*[\frac{\bar{X} + \rho_{xy}}{\bar{x}^* + \rho_{xy}}]$, $t^*{}_3 = \bar{y}^*[\frac{\bar{X} + c_x}{\bar{x}^* + c_x}]$, and $t^*{}_4 = \bar{y}^*[\frac{\bar{X} + \beta_{xy}}{\bar{x}^* + \beta_{xy}}]$,

Where $\bar{x}^*$ and $\bar{y}^*$ are the sample means of the auxiliary variables and variable of interest yet to be drawn with the relationships:

(i) $\bar{X} = f\bar{x} + (1 - f)\bar{x}^*$ and

(ii) $\bar{Y} = f\bar{y} + (1 - f)\bar{y}^*$ [15].

The Mean Square Errors of these estimators $t^*{}_i, i = 1,..,4$ are as follows:

(i) $$mse(t^*{}_1) = (\frac{n}{N-n})^2 mse(t_1) \quad (9)$$

(ii) $$mse(t^*{}_2) = (\frac{n}{N-n})^2 mse(t_2) \quad (10)$$

(iii) $$mse(t^*{}_3) = (\frac{n}{N-n})^2 mse(t_3) \quad (11)$$

(iv) $$mse(t^*{}_4) = (\frac{n}{N-n})^2 mse(t_4) \quad (12)$$

2.3 On the newly Proposed Modified Ratio Estimators

The proposed modified ratio estimators are given as:

$$\bar{y}_{raa1} = \alpha(\frac{\bar{y}}{\bar{x}})(\bar{X} + \rho_{xy}) \quad (13)$$

$$\bar{y}_{raa2} = \alpha(\frac{\bar{y}}{\bar{x}})(\bar{X} + c_x) \quad (14)$$

$$\bar{y}_{raa3} = \alpha(\frac{\bar{y}}{\bar{x}})(\bar{X} + \beta_{xy}) \quad (15)$$

Where $\beta_{xy} = \frac{\rho_{xy} s_x s_y}{s^2{}_x} = \frac{\rho_{xy} s_y}{s_x}$, $\bar{x} = \bar{X}(1 + \Delta_{\bar{x}})$, $\bar{y} = \bar{Y}(1 + \Delta_{\bar{y}})$, $\Delta_{\bar{y}} = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$, $\Delta_{\bar{x}} = \frac{\bar{x} - \bar{X}}{\bar{X}}$ such that $|\Delta_{\bar{y}}| < 1$ and $|\Delta_{\bar{x}}| < 1$

The biases and mean square errors of these proposed modified ratio estimators, $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$ are given as:-

$$bias(\bar{y}_{raa1}) = \alpha(\frac{\bar{X} + \rho_{xy}}{\bar{X}})\bar{Y}(\frac{N-n}{Nn})(c^2{}_x - \rho_{xy} c_x c_y) = \alpha(\frac{\bar{X} + \rho_{xy}}{\bar{X}}) bias(t_1) \quad (16)$$

$$mse(\bar{y}_{raa1}) = \alpha^2 (\frac{\bar{X} + \rho_{xy}}{\bar{X}})^2 \bar{Y}^2 (\frac{N-n}{Nn})[c^2{}_x + c^2{}_y - 2\rho_{xy} c_x c_y] = \alpha^2 (\frac{\bar{X} + \rho_{xy}}{\bar{X}})^2 mse(t_1) \quad (17)$$

$$bias(\bar{y}_{raa2}) = \alpha (\frac{\bar{X} + c_x}{\bar{X}}) \bar{Y} (\frac{N-n}{Nn})(c^2{}_x - \rho_{xy} c_x c_y) = \alpha (\frac{\bar{X} + c_x}{\bar{X}}) bias(t_1) \quad (18)$$

$$mse(\bar{y}_{raa2}) = \alpha^2 (\frac{\bar{X} + c_x}{\bar{X}})^2 \bar{Y}^2 (\frac{N-n}{Nn})[c^2{}_x + c^2{}_y - 2\rho_{xy} c_x c_y] = \alpha^2 (\frac{\bar{X} + c_x}{\bar{X}})^2 mse(t_1) \quad (19)$$

$$bias(\bar{y}_{raa3}) = \alpha (\frac{\bar{X} + \beta_{xy}}{\bar{X}}) \bar{Y} (\frac{N-n}{Nn})(c^2{}_x - \rho_{xy} c_x c_y) = \alpha (\frac{\bar{X} + \beta_{xy}}{\bar{X}}) bias(t_1) \quad (20)$$

$$mse(\bar{y}_{raa3}) = \alpha^2 (\frac{\bar{X} + \beta_{xy}}{\bar{X}})^2 \bar{Y}^2 (\frac{N-n}{Nn})[c^2{}_x + c^2{}_y - 2\rho_{xy} c_x c_y] = \alpha^2 (\frac{\bar{X} + \beta_{xy}}{\bar{X}})^2 mse(t_1) \quad (21)$$

Provided $0 < \alpha < 1$, $(\frac{\bar{X} + \rho_{xy}}{\bar{X}}) \geq 1$, $(\frac{\bar{X} + c_x}{\bar{X}}) \geq 1$ and $(\frac{\bar{X} + \beta_{xy}}{\bar{X}}) \geq 1$.

2.4 Derivation of Biases and Mean Square Errors of $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$

Let,

$\bar{y}_{raa1} = \alpha (\frac{\bar{y}}{\bar{x}})(\bar{X} + \rho_{xy})$, where, $\bar{x} = \bar{X}(1 + \Delta_{\bar{x}})$, $\bar{y} = \bar{Y}(1 + \Delta_{\bar{y}})$, $\Delta_{\bar{y}} = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$, $\Delta_{\bar{x}} = \frac{\bar{x} - \bar{X}}{\bar{X}}$ such that $|\Delta_{\bar{y}}| < 1$ and $|\Delta_{\bar{x}}| < 1$ as earlier defined.

Therefore, using power series expansion,

$$\bar{y}_{raa1} = \alpha (\frac{\bar{y}}{\bar{x}})(\bar{X} + \rho_{xy}) = \alpha (\frac{\bar{Y}(1 + \Delta_{\bar{y}})}{\bar{X}(1 + \Delta_{\bar{x}})})(\bar{X} + \rho_{xy})$$

$$= \alpha \frac{(\bar{X} + \rho_{xy})}{\bar{X}} (\frac{\bar{Y}(1 + \Delta_{\bar{y}})}{(1 + \Delta_{\bar{x}})}) = \alpha \frac{(\bar{X} + \rho_{xy})}{\bar{X}} \bar{Y}(1 + \Delta_{\bar{y}})(1 + \Delta_{\bar{x}})^{-1}$$

$$= \alpha \frac{(\bar{X} + \rho_{xy})}{\bar{X}} \bar{Y}(1 + \Delta_{\bar{y}})(1 - \Delta_{\bar{x}} + \Delta^2{}_{\bar{x}})$$

$$= \alpha \frac{(\bar{X} + \rho_{xy})}{\bar{X}} \bar{Y}(1 - \Delta_{\bar{x}} + \Delta^2{}_{\bar{x}} + \Delta_{\bar{y}} - \Delta_{\bar{x}} \Delta_{\bar{y}} + \Delta^2{}_{\bar{x}} \Delta_{\bar{y}})$$

$$bias(\bar{y}_{raa1}) = \alpha \frac{(\bar{X} + \rho_{xy})}{\bar{X}} E[\bar{Y}(1 - \Delta_{\bar{x}} + \Delta^2{}_{\bar{x}} + \Delta_{\bar{y}} - \Delta_{\bar{x}} \Delta_{\bar{y}} + \Delta^2{}_{\bar{x}} \Delta_{\bar{y}}) - \bar{Y})$$

$$bias(\bar{y}_{raa1}) = \alpha \frac{(\bar{X} + \rho_{xy})}{\bar{X}} \bar{Y} E[\Delta^2_{\bar{x}} - \Delta_{\bar{x}} \Delta_{\bar{y}}]$$, ignoring higher order

Let, $E(\Delta_{\bar{y}}) = E(\Delta_{\bar{x}}) = 0, E(\Delta_{\bar{x}}{}^2) = \frac{S^2{}_x}{\bar{X}^2} = c^2{}_x, E(\Delta_{\bar{y}}{}^2) = \frac{S^2{}_y}{\bar{Y}^2} = c^2{}_y$ and

$E(\Delta_x \Delta_y) = \frac{S_{xy}}{\overline{XY}} = \rho_{xy} c_x c_y$. Then,

$$bias(\bar{y}_{raa1}) = \alpha \frac{(\bar{X} + \rho_{xy})}{\bar{X}} \bar{Y} (\frac{N-n}{Nn})[c^2{}_{\bar{x}} - \rho_{xy} c_{\bar{x}} c_{\bar{y}}]$$ (as in eq. 16)

$$mse(\bar{y}_{raa1}) = E[\alpha \frac{(\bar{X} + \rho_{xy})}{\bar{X}} (\bar{Y}(1 - \Delta_{\bar{x}} + \Delta^2{}_{\bar{x}} + \Delta_{\bar{y}} - \Delta_{\bar{x}} \Delta_{\bar{y}} + \Delta^2{}_{\bar{x}} \Delta_{\bar{y}}) - \bar{Y})]^2$$

$$mse(\bar{y}_{raa1}) = \alpha^2 (\frac{\bar{X} + \rho_{xy}}{\bar{X}})^2 \bar{Y}^2 (\Delta_{\bar{y}} - \Delta_{\bar{x}})^2$$, ignoring higher orders

$$mse(\bar{y}_{raa1}) = \alpha^2 (\frac{\bar{X} + \rho_{xy}}{\bar{X}})^2 \bar{Y}^2 (\frac{N-n}{Nn})(c^2{}_y + c^2{}_y - 2\rho_{xy} c_y c_y)$$ (as in eq. 17)

Similarly,

$$bias(\bar{y}_{raa2}) = \alpha(\frac{\bar{X} + c_x}{\bar{X}})\bar{Y}(\frac{N-n}{Nn})(c^2{}_x - \rho_{xy} c_x c_y) = \alpha(\frac{\bar{X} + c_x}{\bar{X}}) bias(t_1)$$ (as in eq. 18)

$$mse(\bar{y}_{raa2}) = \alpha^2 (\frac{\bar{X} + c_x}{\bar{X}})^2 \bar{Y}^2 (\frac{N-n}{Nn})[c^2{}_x + c^2{}_y - 2\rho_{xy} c_x c_y] = \alpha^2 (\frac{\bar{X} + c_x}{\bar{X}})^2 mse(t_1)$$ (as in eq. 19)

$$bias(\bar{y}_{raa3}) = \alpha(\frac{\bar{X} + \beta_{xy}}{\bar{X}})\bar{Y}(\frac{N-n}{Nn})(c^2{}_x - \rho_{xy} c_x c_y) = \alpha(\frac{\bar{X} + \beta_{xy}}{\bar{X}}) bias(t_1)$$ (as in eq. 20)

$$mse(\bar{y}_{raa3}) = \alpha^2 (\frac{\bar{X} + \beta_{xy}}{\bar{X}})^2 \bar{Y}^2 (\frac{N-n}{Nn})[c^2{}_x + c^2{}_y - 2\rho_{xy} c_x c_y] = \alpha^2 (\frac{\bar{X} + \beta_{xy}}{\bar{X}})^2 mse(t_1)$$

(as in eq. 21).

2.5 Data used

The conditions attached to these modified ratio estimators are that: $0 < \alpha < 1$, $(\frac{\bar{X} + \rho_{xy}}{\bar{X}}) \geq 1$, $(\frac{\bar{X} + c_x}{\bar{X}}) \geq 1$ and $(\frac{\bar{X} + \beta_{xy}}{\bar{X}}) \geq 1$; and $(\frac{\bar{X}}{\bar{X} + \rho_{xy}}) < 1$, $(\frac{\bar{X}}{\bar{X} + c_x}) < 1$ and $(\frac{\bar{X}}{\bar{X} + \beta_{xy}}) < 1$ [1]. To support this claim, these two data sets are used to give room for easy comparison (Table 1).

Table 1: Summary of Data Sets

Population	I	II
Source	Subramani and Kumarapandiyan [12]	Kadilar and Cingi [16]
Case	When $\bar{X} < \bar{Y}$	When $\bar{X} > \bar{Y}$
Population (N)	49	106
Sample (n)	20	20
$\bar{X}$	98.6765	243.76
$\bar{Y}$	116.1633	15.37
c_x	1.0435	2.02
c_y	0.8508	4.18
ρ_{xy}	0.6904	0.82
s_x	102.9709	492.3952
s_y	98.8286	64.2466
β_{xy}	0.6626	0.1069917
$\frac{\bar{X}}{\bar{X}+\rho_{xy}}$	0.9931	0.9966
$\frac{\bar{X}}{\bar{X}+c_x}$	0.9895	0.9918
$\frac{\bar{X}}{\bar{X}+\beta_{xy}}$	0.9933	0.9996
$\frac{\bar{X}+\rho_{xy}}{\bar{X}}$	1.0069966	1.003364
$\frac{\bar{X}+c_x}{\bar{X}}$	1.0157496	1.008287
$\frac{\bar{X}+\beta_{xy}}{\bar{X}}$	1.006714871	1.0004389

3.0 Results and Discussion

The results obtained in this study are presented in Tables 2 to 4. The data showed the mean square errors obtained on Khoshnevisan *et al* [1] ratio estimators (t_0, t_1, t_2, t_3 and t_4), Adewara *et al* [2] modified ratio estimators (t^*_1, t^*_2, t^*_3 and t^*_4) and the proposed modified ratio estimators ($\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$) using Populations I and II (Table 2). The proposed modified ratio estimators, $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$ are found to be better and more efficient than the existing ratio estimators employed in this study with respect to the value of α considered in this study with respect to the two populations (Tables 3 and 4).

Table 2: Mean Square Errors from Some Existing Ratio Estimators [1, 2] using Population I and Population II

Estimator	Mse Population I	Mse Population II
t_0	289.0445	167.4414
t_1	234.3407	73.8414
t_2	231.7389	74.2909
t_3	230.3978	74.0231
t_4	231.8137	73.7766
t^*_1	111.4582	3.9936
t^*_2	110.2206	4.0179
t^*_3	109.5828	4.0036
t^*_4	110.2562	3.9901

Table 3: Mean Square Errors obtained on $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$ using Population I

α	$\bar{y}_{raa1}$	$\bar{y}_{raa2}$	$\bar{y}_{raa3}$
1.0	237.6313	241.7804	237.4982
0.9	192.4814	195.8421	192.3735
0.8	152.0840	154.7395	151.9988
0.7	116.4393	118.4724	116.3741
0.6	85.5473	87.0409	85.4994
0.5	59.4078	60.4461	59.3746
0.4	38.0210	38.6848	37.9997
0.3	21.3868	21.7602	21.3748
0.2	9.5053	9.6712	9.4999
0.1	2.3763	2.4178	2.3750

Table 4: Mean Square Errors obtained on $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$ using Population II

α	$\bar{y}_{raa1}$	$\bar{y}_{raa2}$	$\bar{y}_{raa3}$
1.0	74.3390	75.0701	73.9064
0.9	60.2146	60.8069	59.8642
0.8	47.5770	48.0460	47.3001
0.7	36.4261	36.7844	36.2141
0.6	26.7620	27.0253	26.6064
0.5	18.5848	18.7676	18.4766
0.4	11.9024	12.0112	11.8250
0.3	6.6905	6.7563	6.6516
0.2	2.9737	3.0028	2.9563
0.1	0.7434	0.7507	0.7391

From the estimates in Table 2 for Population I, when $\alpha = 1,$ the ratio estimator, $t^*{}_3$, has the least mse, hence the most preferred and efficient ratio estimator [2]. When $0.1 \le \alpha \le 0.6$, the proposed modified ratio estimators $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$ are better than the existing ratio estimators [1, 2]. However, the proposed modified ratio estimator $\bar{y}_{raa3}$ is most preferred in this range ($0.1 \le \alpha \le 0.6$) because it has the least mse and that $\bar{X} < \bar{Y}$. For Population II, the estimates obtained indicated that when $\alpha = 1,$ ratio estimator, $t^*{}_4$, has the least mse, hence the most preferred and efficient ratio estimator [2].

When $0.1 \le \alpha \le 0.2$, the proposed modified ratio estimators $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$, are better than the ratio estimators used for comparison [1, 2]. As obtained for population I, the proposed modified ratio estimator $\bar{y}_{raa3}$ is the most preferred in this range ($0.1 \le \alpha \le 0.2$) because it has the least mse and that $\bar{X} > \bar{Y}$.

4.0 Conclusion

From all the estimates obtained using both populations I and II, the proposed modified ratio estimators, $\bar{y}_{raa1}$, $\bar{y}_{raa2}$ and $\bar{y}_{raa3}$ whenever $0.1 \le \alpha \le 0.2$, are preferred to other ratio estimators used in this study. However, in Populations I and II, $\bar{y}_{raa3}$ was preferred. Whenever $\bar{X} < \bar{Y}$, the range values of α was found to be wider (i.e. $0.1 \le \alpha \le 0.6$) than when $\bar{X} > \bar{Y}$ (i.e. $0.1 \le \alpha \le 0.2$). Hence, the proposed modified ratio estimators are recommended for usage in sample surveys.

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