Effects of Variable Viscosity on Heat and Mass Transfer over an Exponentially Porous Stretching Surface

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Abstract

This study was conducted to investigate the effects of variable viscosity in boundary layer flow on heat and mass transfer of incompressible viscous fluid over a porous stretching surface in the presence of thermal radiation. The equations governing fluid flow, heat and mass transfer were simplified using similarity variables. Appropriate parameters were introduced to transform the governing equations. The transformation was used to reduce the governing system of Partial differential equations to a set of nonlinear Ordinary Differential Equations, which were then solved numerically using the fourth-order Runge-Kutta method with shooting techniques. The numerical computations for variable fluid parameters controlling fluid flow, heat and mass transfer as well as the effect of varied parameters on velocity, temperature and concentration were presented in tabular and graphical forms. The results showed that an increase in radiation produced a marginal rise on the velocity, temperature and concentration profiles. Skin friction, Nusselt and Sherwood number increased/decreased with varied parameters. It was further observed that increased variation of magnetic parameter ratio led to a corresponding increase in Skin friction, Nusselt number and Sherwood numbers. The results revealed that increase in *Sc*. parameter led to increase in Skin friction, Nusselt and Sherwood numbers.

Keywords: Boundary layer flow, incompressible fluid, Runge-Kutta, Shooting Techniques

1.0 Introduction

The study of variable viscosity on heat and mass transfer over an exponentially porous stretching surface in recent time is of great importance. The enormous contributions of such study in various engineering and industrial processes are significant and diverse. The study is applicable in glass-fibre production, condensation process of metallic plate in cooling bath and aerodynamics extrusions of plastic sheets. In literature, researchers have investigated radiative Magnetohydrodynamics (MHD) flow over a vertical plate with convective boundary conditions where the partial differential equations were transformed into coupled nonlinear differential equations [1]. The numerical results for the skin friction coefficient, the rate of heat transfer represented by local Nusselt number and plate surface temperature were presented and illustrated graphically to interpret the effects. The study also analysed the effects of Biot number, Grashof number, Magnetic field parameter, Eckert number, Prandtl number and radiation parameter.

In the same manner, Hunegnaw and Kishan [2] examined the unsteady MHD heat and mass transfer flow over stretching sheet in a porous medium with variable properties considering viscous dissipation and chemical reactions. The outcome of their study showed that the skin friction coefficient and temperature gradient increase as velocity/suction parameters increase but Sherwood numbers decreases. The effects of Eckert and Prandtl numbers increased the values of skin friction and rate of mass transfer but reduced the temperature gradient. On the other hand, Yirga and Tesfay [3] analyzed the convective heat and mass transfer in Nano fluids through porous media over stretching sheet with viscous dissipation and chemical reaction effects using Keller Method. The emanating numerical solutions obtained for skin friction, Nuselt and Sherwood numbers as well as for velocity, temperature and concentration profiles for selected parameters increase correspondingly. The validation of their study compared favorably with the existing analysis [4].

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In another study, Devi *et al.* [5] focused on radiation and mass transfer effects on MHD boundary layer flow due to an exponentially stretching sheet with heat source and the work compared favourably with several others [4,6,7]. In the same vein, Elbashbeshy *et al.* [8] studied the effect of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over an exponentially stretching surface with suction in the presence of internal heat generation/absorption. The researchers' emerging model equations compared well with earlier studies [4, 6, 9-11]. Within the same period, Hussain and Ahmad [12] analysed the effect of radiation on free convective flow of fluid with variable viscosity from porous vertical plate. The fluid considered is of an optically dense viscous incompressible fluid of temperature-dependent viscosity. The laminar boundary layer equations governing the flow were shown not to be similar. Other researchers dwelled on the effect of hydromagnetic heat and mass transfer flow over an inclined heated surface as well as the effects of variable viscosity on the flow of non-Newtonian fluid through a porous medium in an inclined channel [13, 14].

The main objective of the present study is to investigate the effects of variable viscosity on heat and mass transfer over an exponentially porous stretching surface. The study employed two-dimensional heat and mass transfer of a free convective flow with radiation, concentration and temperature dependent heat source. It included variable viscosity and thermal conductivity of an incompressible fluid in an exponentially porous medium in stretching surface. The analysis of heat and mass transfer of free convective flow with radiation, concentration was carried out using exponentially porous medium. To achieve the aim of this study, the researcher transforms the continuity, momentum, temperature and concentration equations using dimensionless parameters to obtain the nonlinear differential equations governing the problem; thereby using Runge-Kutta Method with shooting techniques to solve the modelled equations.

2.0 Materials and Methods

In this study, the effects of variable viscosity on heat and mass transfer of incompressible fluid over an exponentially porous stretching surface was performed. The emerging slit at this instance was assumed moving with non-uniform velocity in the presence of thermal radiation and this guided the governing equations. The governing equations of such flow are presented in usual notations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u\frac{\partial}{\partial y}\left(\mu(\tau)\frac{\partial u}{\partial y}\right)-\sigma B_0^2 u+g\beta(T-T_\infty)+g\beta_c(C-C_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho cp}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho cp}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Here, *u* and *v* are the components of the velocity in the directions of x and y respectively. $(\mu(\tau))$ is the variable fluid viscosity, ρ is the fluid density, T is the temperature, k is the thermal conductivity of the fluid, β is the volumetric coefficient of thermal expansion, g is the gravity field, T_∞ is the temperature at infinity. Using Rosseland approximation for radiation [15, 16] it can be written as $q_r=-\frac{4\sigma_1}{3R}\frac{\partial T^4}{\partial y}$ where σ is the Stefan-Beltzman constant, R is the absorption coefficient. Assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor's series and expanding T^4 and T_∞ and neglecting higher orders, we get $T^4=4T_\infty^3-3T_\infty^4$ Therefore, the equation (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho cp}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma_1}{3\rho cpR}\frac{\partial q_r}{\partial y} \tag{5}$$

The appropriate boundary conditions for the problems based on the study are given as

$$u = u_0 \exp\left(\frac{x}{l}\right), \quad v = -v_0 \exp\left(\frac{x}{l}\right), \quad T = T_w, \; C = C_\infty \quad y = 0 \tag{6}$$

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ as } \quad y \to \infty \tag{7}$$

Where $u(x)$ is the stream-wise velocity and $v(x)$ is the velocity of the suction of the fluid, T_w is the wall temperature. In order to solve the formulated problem, the following similarity variable relations for u, v were introduced as equation (8) where $\psi(x, y)$ is the stream function.

$$u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}, \tag{8}$$

In order to transform equations 1-4, the following dimensionless parameters were defined as:

$$\theta(\eta) = \frac{T - T_\infty}{T_\omega - T_\infty}, \; \phi(\eta) = \frac{C - C_\infty}{C_\omega - C_\infty}, \; \eta = \frac{y}{l}\sqrt{\frac{\mathrm{Re}}{2} e^{\frac{x}{2l}}}, T = T_\infty + (T_w - T_\infty) e^{\frac{2x}{l}} \theta(\eta),$$

$$C = C_\infty + (C_w - C_\infty) e^{\frac{2x}{l}} \phi(\eta) \tag{9}$$

Substituting equation (8) and (9) into equations (1)-(4), the equations (10)-(12) after some manipulations using appropriate boundary conditions, the following modelled equations emerge.

$$(1 + \gamma(1-\theta))\left(\frac{d^3}{d\eta^3} f(\eta)\right) - \gamma\left(\frac{d}{d\eta}\theta(\eta)\right)\left(\frac{d^2}{d\eta^2} f(\eta)\right) + \frac{1}{2} f(\eta)\left(\frac{d^2}{d\eta^2} f(\eta)\right) - 2\left(\frac{d}{d\eta} f(\eta)\right)^2 - M\left(\frac{d}{d\eta} f(\eta)\right) + G\gamma\theta(\eta) = 0, \tag{10}$$

$$\left(1 + \frac{4}{3N}\right)\left(\frac{d^2}{d\eta^2}\theta(\eta)\right) + Pr f(\eta)\left(\frac{d}{d\eta}\theta(\eta)\right) = 0 \tag{11}$$

$$\frac{d^2}{d\eta}\phi(\eta) + Scf(\eta)\left(\frac{d}{d\eta}\phi(\eta)\right) = 0 \tag{12}$$

Using appropriate boundary conditions with relevant parameters as in (13), equations (10)-(12) can be arrived at and the results were presented where

$$f' = \frac{df}{d\eta} = 1, \; \frac{df}{d\eta} = s \quad \theta = 1 \quad at \quad y = 0, \quad \frac{df}{d\eta} \to 0, \quad \theta \to 0 \quad as \quad y \to \infty$$

$$N = \frac{Gc}{Gr}, \quad R = \frac{Gr}{\mathrm{Re}^2}, \quad \mathrm{Pr} = \frac{\upsilon}{Cp}, \quad Sc = \frac{\upsilon}{D}, \quad \gamma = \frac{2L\lambda}{U_0} e^{\frac{-x}{L}}, \quad M = \frac{2\sigma B_0^2}{\rho v} e^{\frac{x}{2L}} \tag{13}$$

The equations (10) – (12) along with boundary conditions were solved by converting them to an initial value problem as in (14)-(17). Therefore, let

$$f' = z, z' = p, \theta' = q \tag{14}$$

$$p' = -\gamma f p - \frac{1}{2} + 2z^2 + Mq - G\gamma q\theta \tag{15}$$

$$q' = -\frac{\Pr fq}{\left(1+\frac{4}{3N}\right)} \tag{16}$$

$$h' = -Scfh \tag{17}$$

To integrate equations (14) to (17) as an initial value problem requires values for $f''(0)$, $\theta'(0)$ and $\phi'(0)$ but no such values were given in the boundary conditions. Therefore, the suitable guess values for $f''(0)$, $\theta'(0)$ and $\phi'(0)$ were chosen and then integrations were carried out. In order to ensure a level of accuracy, the calculated values for $f''(0)$, $\theta'(0)$ and $\phi'(0)$ at $\eta = 4$ with the given boundary conditions $f'(4) = 0$, $\theta(4)$ and $\phi(4)$ and adjusted the estimated values, $f''(0)$, $\theta'(0)$ and $\phi'(0)$, to give a better approximation for the solutions were carried out. The series of values for $f''(0)$, $\theta'(0)$ and $\phi'(0)$, were performed, then a fourth-order classical Runge–Kutta method with shooting techniques with step-size $h = 0.01$ were used. The above procedure was repeated until the results up to the desired degrees of accuracy 10^{-5} were obtained.

3.0 Results and Discussion

The velocity profiles for variation of γ, M and G parameters are presented in Tables 1, 2 and 3 respectively. Table 1 indicates that the variation of chemical reaction parameter led to the variation/fluctuation in skin friction, but the increase in the same parameter transforms into marginal increase in Nusselt and Sherwood numbers respectively. The variations of this parameter suggested corresponding effects on the rate of heat and mass transfer of particles. The effect of Magnetic parameter on Skin friction, Nusselt number and Sherwood number presents some variations (Table 2). It was discovered that increased variation of magnetic parameter ratio led to a corresponding increase in Skin friction, Nusselt number and Sherwood number respectively. The outcome revealed that there was increase in heat and mass transfer. With respect to the effect of acceleration parameter on Skin friction, Nusselt Number and Sherwood Number, it was discovered that the marginal increase in this parameter led to increase in Skin friction, rise in Nusselt and Sherwood numbers respectively (Table 3). In essence, the acceleration parameter gave rise to heat and mass transfer.

Table 1: Velocity Profile for Variation of γ Parameter

γ	***M***	***G***	***Pr.***	***Sc.***	***N***	$f^{11}(0)$	$-\theta^1(0)$	$-\phi^1$ **(0)**
-0.1	0.1	0.5	0.71	3	5	-1.4579	0.6106	2.2593
0.0	0.1	0.5	0.71	3	5	-1.3770	0.6181	2.2593
5	0.1	0.5	0.71	3	5	-1.4575	0.7076	2.3877
7	0.1	0.5	0.71	3	5	-0.3712	0.7167	2.3985

Table 2: Velocity Profile for Variation of *M* Parameter

γ	***M***	***G***	***Pr.***	***Sc.***	***N***	$f^{11}(0)$	$-\theta^1(0)$	$-\phi^1$ **(0)**
0.1	0.1	0.5	0.71	3	5	-1.4579	0.6106	2.2593
0.1	0.3	0.5	0.71	3	5	-1.4555	0.6107	2.2593
0.1	0.5	0.5	0.71	3	5	-0.5217	0.7002	2.3793
0.1	0.7	0.5	0.71	3	5	-0.4514	0.7074	2.3881

Table 3: Velocity Profile for Variation of *G* Parameter

γ	***M***	***G***	***Pr***	***Sc***	***N***	$f^{11}(0)$	$-\theta^{1}(0)$	$-\phi^{1}$ **(0)**
0.1	0.1	0.5	0.71	3	5	-1.4579	0.6106	2.2593
0.1	0.1	0.6	0.71	3	5	-1.3770	0.6181	2.2701
0.1	0.1	0.7	0.71	3	5	-0.3369	0.7187	2.4020
0.1	0.1	0.8	0.71	3	5	-0.1742	0.7343	2.4212

Table 4 describes the effect of Prandtl number parameter on Skin friction, Nusselt number and Sherwood number. It was discovered that increase in Prandtl led to increase in Skin friction, Nusselt number and Sherwood number respectively. The significance of this outcome suggested that increase in *Pr*. gave rise to the high temperature and chemical reaction. Table 5 presents the effect of varying *Sc*. on Skin friction, Nusselt number and Sherwood number. The results revealed that increase in *Sc*. parameter led to increase in Skin friction, Nusselt and Sherwood numbers respectively. Table 6 reports the variations of *N* parameters on Skin friction, Nusselt number and Sherwood number. An increase in this parameter led to increase in skin friction whereas its marginal increase led to corresponding marginal increase in Nusselt number and Sherwood number. Figs. 1-6 depict the flow of the fluids and the effects on the skin friction, Nusselt number and Sherwood number with respect to the different fluid parameters.

Table 4: Temperature Profile for Variation of *Pr*, Parameter

γ	***M***	***G***	***Pr***	***Sc***	***N***	$f^{11}(0)$	$-\theta^{1}(0)$	$-\phi^{1}$ **(0)**
0.1	0.1	0.5	0.71	3	5	-1.4579	0.6106	2.2593
0.1	0.1	0.5	0.73	3	5	-1.3770	0.6297	2.2701
0.1	0.1	0.5	0.75	3	5	-0.4680	0.7328	2.3863
0.1	0.1	0.5	0.77	3	5	-0.3877	0.7547	2.3964

Table 5: Concentration Profile for Variation of *Sc*. Parameter

γ	***M***	***G***	***Pr***	***Sc***	***N***	$f^{11}(0)$	$-\theta^{1}(0)$	$-\phi^{1}$ **(0)**
0.1	0.1	0.5	0.71	3	5	-1.4579	0.6106	2.2593
0.1	0.1	0.5	0.71	5	5	-1.3770	0.6181	3.4514
0.1	0.1	0.5	0.71	7	5	-0.4575	0.7076	4.6770
0.1	0.1	0.5	0.71	9	5	-0.3712	0.7167	5.7719

Table 6 Temperature Profile for Variation of *N* Parameter

γ	***M***	***G***	***Pr***	***Sc***	***N***	$f^{11}(0)$	$-\theta^{1}(0)$	$-\phi^{1}$ **(0)**
0.1	0.1	0.5	0.71	3	5	-1.4519	0.6106	2.2593
0.1	0.1	0.5	0.71	3	10	-1.3770	0.6667	2.2701
0.1	0.1	0.5	0.71	3	15	-0.4870	0.7803	2.3839
0.1	0.1	0.5	0.71	3	25	-0.4094	0.8072	2.3936

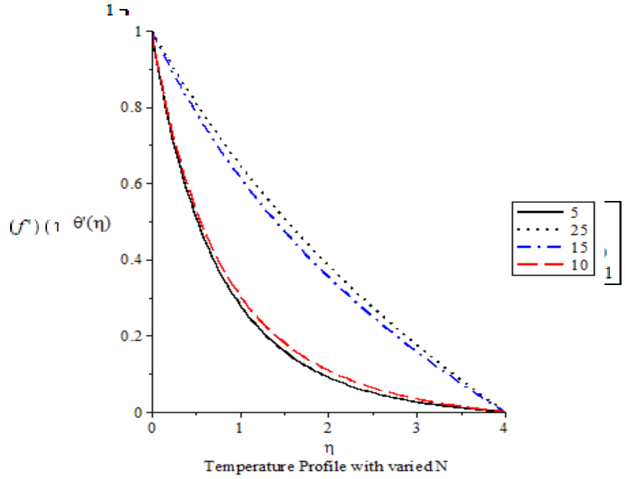


Figure 1: Velocity profile with varied γ

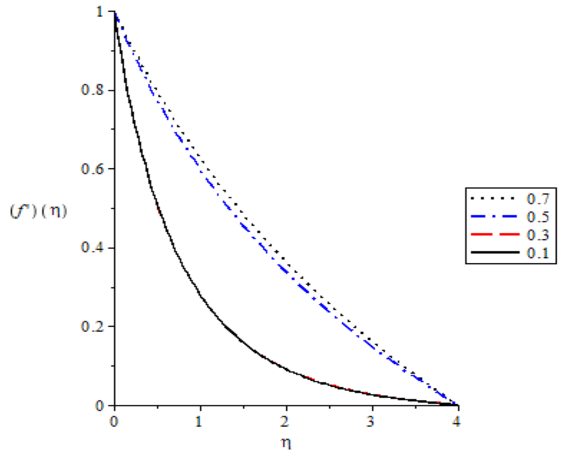


Figure 2: Velocity profile with varied N

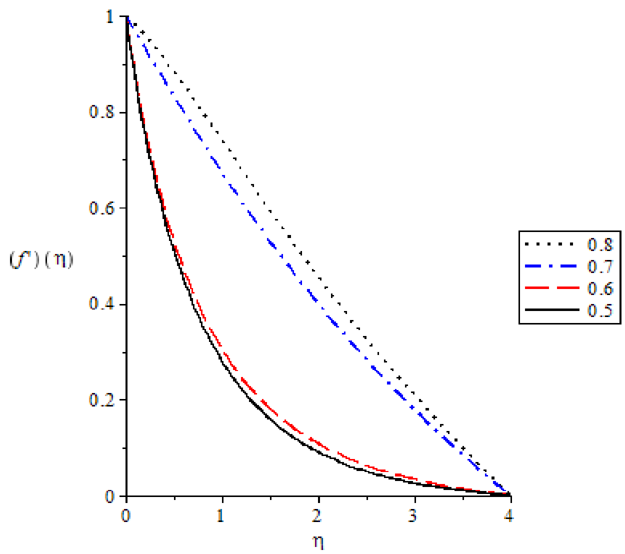


Figure 3 : Velocity Profile with varied G

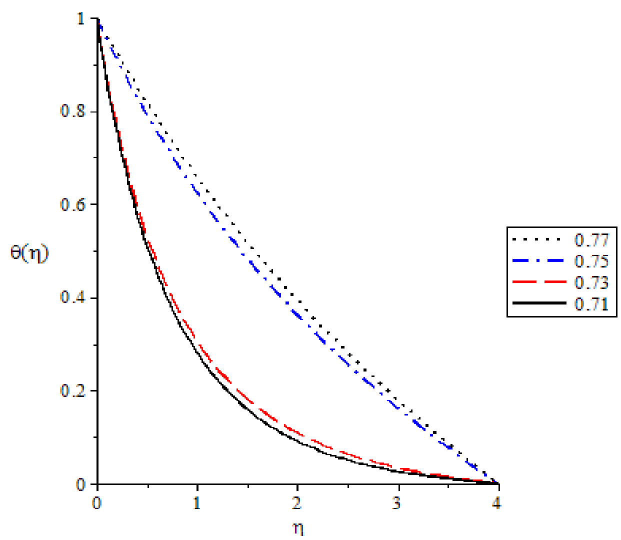


Figure 4 Temperature Profile with varied Pr.

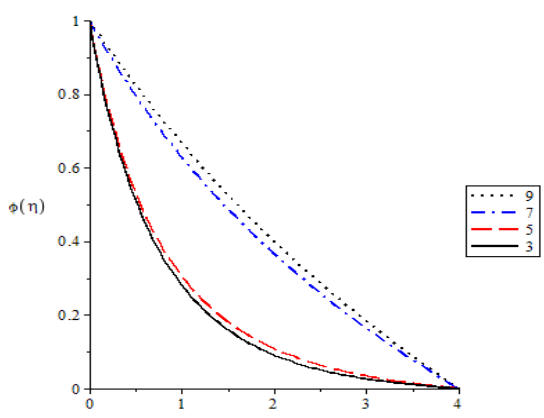
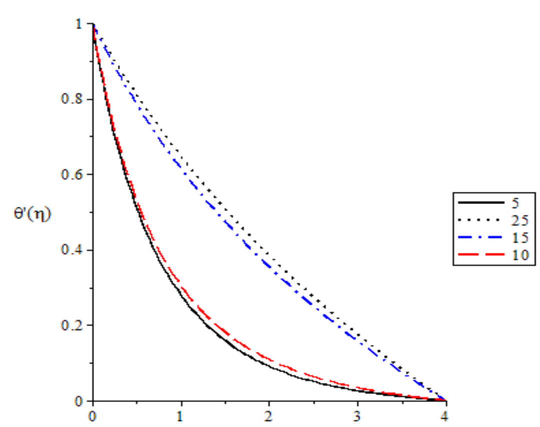


Figure 5 Concentration profile for varied Sc,



Temperature Profile with varied N

The present study considered the effect of variable viscosity on the heat and mass transfers over an exponentially porous stretching surface. The effect of chemical reaction, magnetic, acceleration, Prandtl, mass diffusion/viscosity and radiation parameters on skin friction, Nusselt number and Sherwood numbers was presented. Considering the nature of the study, numerical solutions of the modelled equations were performed for various fluid parameters controlling the fluid flow, heat and mass transfer. The interesting features of significant parameters on velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number were highlighted. It was discovered that when all parameters used in the investigations are varied, there are marginal correspondence variations (increase/decrease) in skin friction, Nusselt number and Sherwood number.

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